Building Bridges

Book of abstracts

3rd EU/US Workshop on
Automorphic Forms and Related Topics

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Organizers:
Jay Jorgenson (New York)
Lejla Smajlovic (Sarajevo)
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Abelian varieties and the inverse Galois problem

Samuele Anni

Abstract

Let $\overline{\mathbb{Q}}$ be an algebraic closure of $\mathbb{Q}$, let n be a positive integer and let ℓ a prime number. Given a curve C over $\mathbb{Q}$ of genus g, it is possible to define a Galois representation $\rho : \mathrm{Gal}(\overline{\mathbb{Q}}/\mathbb{Q}) \to \mathrm{GSp}_{2g}(\mathbb{F}_\ell)$, where $\mathbb{F}_\ell$ is the finite field of ℓ elements and GSp_{2g} is the general symplectic group in GL_{2g}, corresponding to the action of the absolute Galois group $\mathrm{Gal}(\overline{\mathbb{Q}}/\mathbb{Q})$ on the ℓ-torsion points of its Jacobian variety $J(C)$. If ρ is surjective, then we realize $\mathrm{GSp}_{2g}(\mathbb{F}_\ell)$ as a Galois group over $\mathbb{Q}$. In this talk I will first describe a joint work with Pedro Lemos and Samir Siksek, concerning the realization of $\mathrm{GSp}_6(\mathbb{F}_\ell)$ as a Galois group for infinitely many odd primes ℓ and then the generalizations to arbitrary dimensions, joint work with Vladimir Dokchitser.

Dedekind sums for cofinite Fuchsian groups

Claire Burrin

Abstract

Dedekind sums are very well-studied arithmetic sums, which determine the modular transformation of the eta-function. We present the construction of the general analogue of the Dedekind sums, which we call Dedekind symbol, for cofinite Fuchsian groups. If time permits, we will discuss certain aspects of the Dedekind symbol, such as its relation to Selberg–Kloosterman sums, the equidistribution mod 1 of its values, and the existence of a reciprocity formula.

Period integrals and special values of L-functions

Andrew Corbett

Abstract

In the theory of L-functions, special values are frequently shown to equal mysterious arithmetic objects. I will talk about some recent results on special values of automorphic L-functions in the spirit of the Gan–Gross–Prasad conjectures. Our mysterious arithmetic objects are period integrals of automorphic forms on both GL(2) and GSp(4). The applications are surprising and range from non-vanishing problems to bounding Lp-norms of modular forms.

Refinements of Boecherer's conjecture

Martin Dickson

Abstract

In the 1980s Boecherer made a remarkable conjecture relating the Fourier coefficients of a Siegel modular form of degree two to special values of the twists of its spin L-function. I will talk about refinements of this conjecture, and attempts to computationally verify these, which follow from an explicit form of the Gan–Gross–Prasad conjectures. Part of this is joint work with A. Pitale, A. Saha, and R. Schmidt.

Torsion subgroups of elliptic curves over elementary Abelian extensions

Ozlem Ejder

Abstract

Let K denote the quadratic field $\mathbb{Q}(\sqrt{d})$ where $d = -1$ or -3. Let E be an elliptic curve defined over K. In this paper, we analyze the torsion subgroups of E in the maximal elementary abelian 2-extension of K.

Selmer groups and Beilinson-Bloch-Kato conjectures

Yara Elias

Abstract

Kolyvagin's method of Euler systems is used to bound Selmer groups associated to elliptic curves defined over $\mathbb{Q}$ and certain imaginary quadratic fields assuming the non-vanishing of a suitable Heegner point. Its adaptations due to Nekovar, and Bertolini and Darmon allow us to bound Selmer groups associated to certain modular forms twisted by algebraic Hecke characters assuming the non-vanishing of a suitable Heegner cycle. In this talk, we will discuss the contributions of these results to the Birch-Swinnerton-Dyer conjecture and the Beilinson-Bloch-Kato conjecture, respectively.

Aspects of the theta correspondence

Jens Funke

Abstract

Theta series associated to quadratic forms are a very classical way for constructing automorphic forms. In this overview talk, we discuss some of the underlying representation-theoretic aspects.

Integral representations by sums of polygonal numbers

Anna Haensch

Abstract

In this talk we will consider representations by ternary sums of polygonal numbers. A sum of three polygonal numbers is a ternary inhomogeneous quadratic polynomial, and by completing the square the representation problem for such a ternary sum can be easily recast as a representation problem for a coset of some quadratic lattice. Using the machinery of quadratic lattices, we will consider when a sum of three polygonal numbers represents all but finitely many positive integers.

Degenerate Eisenstein series for GL_n and some applications in number theory

Marcela Hanzer

Abstract

In this talk we will discuss degenerate Eisenstein series for adelic general linear groups; its poles and images. We will apply these results to some classical Eisenstein series. We will also briefly discuss similar problems for classical groups. Most of the talk is joint work with Goran Muić.

Construction of Jacobi and Siegel cusp forms

Abhash Jha

Abstract

Given a fixed Jacobi cusp form, we consider a family of linear maps constructed using Rankin-Cohen brackets and compute the adjoint of these linear maps with respect to Petersson scalar product. The Fourier coefficients of the Jacobi cusp form constructed using this method involve special values of certain Dirichlet series of Rankin type associated to Jacobi forms. We also discuss the similar construction in the case of Siegel modular forms of genus two. This is joint work with B. Sahu.

Regularized determinant of the Lax-Phillips operator and central value of the scattering determinant

Jay Jorgenson

Abstract

Using the superzeta function approach due to Voros, we define a Hurwitz-type zeta function $\zeta_B^{\pm}(s,z)$ constructed from the resonances associated to $zI - [(1/2)I \pm B]$, where B is the Lax-Phillips scattering operator. We prove the meromorphic continuation in s of $\zeta_B^{\pm}(s,z)$ and, define a determinant of the operators $zI - [(1/2)I \pm B]$. We obtain expressions for Selberg's zeta function and the determinant $\phi(s)$ of the scattering matrix in terms of the operator determinants. Using this result, we evaluate $\phi(1/2)$ for any finite volume surface.

Weyl group multiple Dirichlet series and a metaplectic Eisenstein series on $\mathrm{GL}(3,\mathbb{R})$

Edmund Karasiewicz

Abstract

The theory of Weyl group multiple Dirichlet series (Dirichlet series in several variables with a group of functional equations isomorphic to a Weyl group) has been developed by Brubaker, Bump, Chinta, et al. These multiple Dirichlet series are conjectured to be the Fourier coefficients of metaplectic Eisenstein series. The work of Brubaker, Bump, Chinta, et al. includes a hypothesis that forces the base field to be totally imaginary. We will consider a specific metaplectic Eisenstein series over $\mathbb{R}$ and see how this fits into the existing theory.

Diophantine quadruples over finite fields

Matija Kazalicki

Abstract

A Diophantine m-tuple is a set of m positive integers with the property that the product of any two of its distinct elements increased by 1 is a perfect square, e.g.. $\{1, 3, 8, 120\}$ is a Diophantine quadruple. While the problems related to these sets and their rational counterparts have a long history starting from Diophantus, a Diophantine m-tuples with elements in finite fields have not been studied a lot.

In this talk, we will present the formula for the number of Diophantine quadruples in the finite field with p elements (p is a prime). Surprisingly, Fourier coefficients of some modular forms appear in the formula. This is a joint work with Andrej Dujella.

Equidistribution of shears and applications

Dubi Kelmer

Abstract

A "shear" is a unipotent translate of a cuspidal geodesic ray in the quotient of the hyperbolic plane by a non-uniform discrete group (possibly of infinite covolume). In joint work with Alex Kontorovich, we prove the equidistribution of shears under large translates. We give applications including to moments of GL(2) automorphic L-functions, and to counting integer points on affine homogeneous varieties. No prior knowledge of these topics will be assumed.

Periods of modular forms and identities between Eisenstein series

Kamal Khuri-Makdisi

Abstract

Borisov and Gunnells observed in 2001 that certain linear relations between products of two holomorphic weight 1 Eisenstein series had the same structure as the relations between periods of modular forms; a similar phenomenon exists in higher weights. We give a conceptual reason for this observation in arbitrary weight. This involves an unconventional way of expanding the Rankin-Selberg convolution of a cusp form with an Eisenstein series.

On arbitrary products of eigenforms

Arvind Kumar

Abstract

We characterize all the cases in which products of arbitrary numbers of nearly holomorphic eigenforms and products of arbitrary numbers of quasimodular eigenforms for the full modular group $\mathrm{SL}_2(\mathbb{Z})$ are again eigenforms.

Models for modular curves $X_0(N)$

Iva Kodrnja

Abstract

The classical modular curve $X_0(N)$ is defined to be the quotient of the extended upper half-plane $\mathbb{H}^* = \{z \in \mathbb{C} : \mathrm{Im}(z) > 0\} \cup \mathbb{Q} \cup \{\infty\}$ by the action of congruence subgroup $\Gamma_0(N)$. It has the structure of a compact Riemann surface and thus can be interpreted as smooth irreducible projective algebraic curve.

The canonical equation of $X_0(N)$ (also called modular equation) is hard to compute and has several practical drawbacks. In his doctoral thesis, S. Galbraith has presented a method for finding defining equations of $X_0(N)$ based on canonical embedding. He used cusp forms of weight 2 to define maps from $X_0(N)$ to projective space. Recently, G. Muić has extended this method and maps $X_0(N)$ to projective plane using modular forms of higher weight.

In this talk I will present results from my doctoral thesis which concerns with Muić's method. Some of the main topics are the choice of appropriate modular forms and computation of the degree of the model. I will give a few examples of models obtained in this way using η-quotients.

Selberg trace formula as a Dirichlet series

Min Lee

Abstract

We explore the idea of Conrey and Li of presenting the Selberg trace formula for Hecke operators, as a Dirichlet series. We explore the idea of Conrey and Li presenting the Selberg trace formula for Hecke operators, as a Dirichlet series. We enhance their work in few ways and present several applications of our formula. This is a joint work with Andrew Booker.

A lower bound for the least prime in an arithmetic progression

Junxian Li

Abstract

Fix k a positive integer, and let ℓ be coprime to k. Let $p(k,\ell)$ denote the smallest prime equivalent to $\ell \pmod{k}$, and set $P(k)$ to be the maximum of all the $p(k,\ell)$. We seek lower bounds for $P(k)$. In particular, we show that for almost every k one has $P(k) \gg \phi(k) \log k \log_2 k \log_4 k / \log_3 k,$. This is a joint work with K. Pratt and G. Shakan based on the recent work on large gaps between primes of of Ford, Green, Konyangin, Maynard, and Tao.

Bounds on the arithmetic genus of Hilbert modular varieties

Benjamin Linowitz

Abstract

Associated to every totally real number field k of degree n is a Hilbert modular variety which arises as a desingularization of the compactification of the quotient space of the Hilbert modular group acting on an n-fold product of complex upper half planes. In this talk we will discuss some recent results concerning the arithmetic genus of Hilbert modular varieties. In particular we will exhibit upper and lower bounds for the arithmetic genus of Hilbert modular varieties in terms of the degree and discriminant of k. Additionally, we will discuss some recent progress on the related problem of classifying those totally real fields k for which the associated Hilbert modular variety is not of general type.

Using theta lifts to understand Eisenstein denominators

Robert Little

Abstract

We use the (extended) Shimura-Shintani correspondence (in the cohomological setting of Funke-Millson) to investigate Harders theory of denominators of Eisenstein cohomology classes on modular curves.

Fourier coefficients of automorphic forms for higher rank groups

Guangshi Lyu

Abstract

Fourier coefficients of automorphic forms are interesting and important objects in modern number theory. In this talk, I shall introduce some recent progress on orthogonality between additive characters and Fourier coefficients of cusp forms over primes. If time permits, I shall also talk about shifted convolution sums for higher rank groups. This talk is based on my recent work joint with Fei Hou and Yujiao Jiang.

Zeros of L-functions of certain half integral weight cusp forms

Jaban Meher

Abstract

We will discuss about the zeros of L-functions attached to half integral weight cusp forms for the group $\Gamma_0(4)$.

Computing the p-adic periods of abelian surfaces from automorphic forms

Marc Masdeu

Abstract

Let F be a number field, and let f be a normalized eigenform modular form of weight 2 and level N for GL(2,F). It is conjectured that there is an abelian variety A_f attached to such a modular form. This abelian variety should have dimension equal to the degree of the field of Hecke eigenvalues, and should have good reduction outside N. In those instances where the Eichler-Shimura construction is not available (for example when F is not totally-real) little is known about how to find A_f.

In joint work with Xavier Guitart, we present a p-adic conjectural construction (subject to several restrictions, in particular p should divide N) of A_f, and illustrate how in favourable situations it can be used to find equations for abelian surfaces A_f as jacobians of hyperelliptic curves.

Properties of Sturm's operator

Kathrin Maurischat

Abstract

For non-holomorphic modular forms, the projection to their holomorphic components is described by Sturm's operator in all cases known to us. We show that there are interesting situations in which its image fails to be holomorphic, for example the Siegel case of genus two and the low weight three.

Lower bounds on dimensions of mod-p Hecke algebras

Anna Medvedovsky

Abstract

In 2012, Nicolas and Serre revived interest in the study of mod-p Hecke algebras when they proved that, for $p = 2$, the Hecke algebra acting on level-one forms of all weights is the power-series ring F2[[T3, T5]]: a surprising and surprisingly explicit result. Their elementary arguments do not appear to generalize directly to other primes, but their tools — the Hecke recursion, the nilpotence filtration — serve as the backbone of a new method, uniform and entirely in characteristic p, for obtaining lower bounds on dimensions of mod-p Hecke algebras. I will present this new method (currently implemented in the genus-zero case), compare it with others, and discuss various future directions. The talk should be accessible to anyone with some familiarity with modular forms and Hecke operators. The key technical result is pure algebra, combinatorial in flavor; and may be of independent interest.

Numerical computation of generalized τ-Li coefficients

Almasa Odžak

Abstract

The importance of generalized Li and τ-Li coefficients comes from the criteria relating generalized Riemann or tau-Riemann hypothesis for an L-function with certain properties of these coefficients attached to that L-function. An example is the Li criterion for the Riemann hypothesis stating that the Riemann hypothesis is equivalent to non-negativity of corresponding Li coefficients. Some later results include asymptotic behavior of these coefficients.

There are a number of approaches to the problem of numerically computing the Li and τ-Li coefficients of a given function. Computationally speaking, the problem is quite demanding. The main issue is the accumulation of the error terms and the time required for the computation. Arb, a C library for arbitrary-precision floating-point ball arithmetic with automatic error control, turns out to be very useful tool.

Our investigations are focused on two very broad classes of functions $S^{\sharp\flat}(\sigma_0;\sigma_1)$ and $S^{\sharp}_{R}$. Both classes are interested since they contain functions that are assumed to violate Riemann hypothesis in which case numerical evaluation of τ-Li coefficients may serve as a tool for proving existence of eventual zeros off the critical line and eventual zeros off the critical strip.

Rational points on twisted modular curves

Ekin Ozman

Abstract

Studying rational points on an algebraic curve is one of the main problems in number theory. In this talk, I will present results about points on certain twists of the classical modular curve. Many of these twisted curves violate the Hasse principle. In the cases that genus is bigger than one, some of these violations are explained by the Mordell-Weil sieve method which will be defined. In genus one cases, it is possible to study Hasse principle violations via local-global trace obstructions.

An analytic class number type formula for $\mathrm{PSL}_2(\mathbb{Z})$

Anna von Pippich

Abstract

For any Fuchsian subgroup $\Gamma \subset \mathrm{PSL}_2(\mathbb{R})$ of the first kind, Selberg introduced the Selberg zeta function in analogy to the Riemann zeta function using the lengths of simple closed geodesics on $\Gamma\backslash\mathbb{H}$ instead of prime numbers. In this talk, we report on a generalized Riemann–Roch isometry in Arakelov geometry. As arithmetic application, we determine the special value at $s = 1$ of the derivative of the Selberg zeta function in the case $\Gamma = \mathrm{PSL}_2(\mathbb{Z})$. This is joint work with Gerard Freixas.

Elliptic curves with maximally disjoint division fields

James Ricci

Abstract

One of the many interesting algebraic objects associated to a given elliptic curve, E, defined over the rational numbers, is its full-torsion representation $\rho_E : \mathrm{Gal}(\bar{\mathbb{Q}}/\mathbb{Q}) \to \mathrm{GL}_2(\hat{\mathbb{Z}})$. Generalizing this idea, one can create another full-torsion Galois representation, $\rho_{(E_1,E_2)} : \mathrm{Gal}(\bar{\mathbb{Q}}/\mathbb{Q}) \to \left(\mathrm{GL}_2(\hat{\mathbb{Z}})\right)^2$ associated to a pair (E_1, E_2) of elliptic curves defined over $\mathbb{Q}$. The goal of this talk is to provide an infinite number of concrete examples of pairs of elliptic curves whose associated full-torsion Galois representation $\rho_{(E_1,E_2)}$ has maximal image.

Integers that can be written as the sum of two cubes

Eugenia Rosu

Abstract

The Birch and Swinnerton-Dyer conjecture predicts that we have non-torsion rational points on an elliptic curve iff the L-function corresponding to the elliptic curve vanishes at 1. Thus BSD predicts that a positive integer N is the sum of two cubes if $L(E_N, 1) = 0$, where $L(E_N, s)$ is the L-function corresponding to the elliptic curve $E_N : x^3 + y^3 = N$. Using methods from automorphic forms, we have computed several formulas that relate $L(E_N, 1)$ to the trace of a ratio of specific theta functions at a CM point. This offers a criterion for when the integer N is the sum of two cubes. Furthermore, when $L(E_N, 1)$ is nonzero we get a formula for the number of elements in the Tate-Shafarevich group and show that it is a square in certain cases.

Convolution sums of the divisor functions and the number of representations of certain quadratic forms

Brundaban Sahu

Abstract

We report about our recent results on the evaluation of some convolution sums of the divisor functions and their use in determining the number of representations of certain quadratic forms. By comparing with similar results, we also make some observations about the Fourier coefficients of certain cusp forms. This is a joint work with B. Ramakrishnan.

On the Weyl law for certain quantum graphs

Lamija Šćeta

Abstract

We apply a special case of Tauberian theorem for the Laplace transform to the suitably transformed trace formula in the setting of quantum graphs with general self adjoint boundary conditions and obtain new bounds for the remainder term in the Weyl law.

Dimension formulas for Hilbert modular forms and Shimizu L-series

Fredrik Stromberg

Abstract

I will discuss some recent results on joint work with N.-P. Skoruppa, regarding explicit methods, in particular dimension formulas, for vector-valued Hilbert modular forms.

The Hauptmodul at elliptic points of certain arithmetic groups

Holger Then

Abstract

Let N be a square-free integer such that the arithmetic group $\Gamma_0(N)^+$ has genus zero; there are 44 such groups. Let j_N denote the associated Hauptmodul normalized to have residue equal to one and constant term equal to zero in its q-expansion. In joint work together with Lejla Smajlović and Jay Jorgenson, we prove that the Hauptmodul at any elliptic point of the surface associated to $\Gamma_0(N)^+$ is an algebraic integer. Moreover, for each such N and elliptic point e, we explicitly evaluate $j_N(e)$ in radicals and construct generating polynomials of the corresponding class fields of the orders associated to the elliptic point under consideration.

The sign changes of Fourier coefficients of Eisenstein series

Lola Thompson

Abstract

We study a variety of statistical questions concerning the signs of the Fourier coefficients of Eisenstein series, proving analogues of several well-known theorems for cusp forms. This talk is based on joint work with Benjamin Linowitz.

Elliptic and hyperbolic Eisenstein series as theta lifts

Fabian Völz

Abstract

Generalising the concept of classical non-holomorphic Eisenstein series associated to cusps, one can define elliptic Eisenstein series associated to points in the upper-half plane $\mathbb{H}$, and hyperbolic Eisenstein series associated to geodesics in $\mathbb{H}$. In my talk I will show that averaged versions of these elliptic and hyperbolic Eisenstein series can be obtained as Theta lifts of signature $(2, 1)$ of some weighted Poincaré series. These averaged Eisenstein series can also be regarded as traces of elliptic and hyperbolic kernel functions.

On the distribution of Hecke eigenvalues for $GL(2)$

Nahid Walji

Abstract

Given a self-dual cuspidal automorphic representation for GL(2) over a number field, we establish the existence of an infinite number of Hecke eigenvalues that are greater than an explicit positive constant, and an infinite number of Hecke eigenvalues that are less than an explicit negative constant. This provides an answer to a question of Serre. We also consider analogous problems for cuspidal automorphic representations that are not self-dual.

On the lifting of Hilbert cusp forms

Shunsuke Yamana

Abstract

Starting from a Hilbert cusp form, I will construct holomorphic cusp forms on certain Hermitian symmetric domains of higher degree. In the case of Siegel cusp forms, this is a generalization of the Saito-Kurokawa lifting from degree two to higher degrees and the Ikeda lifting from the rational number field to totally real number fields.

This is a joint work with Tamotsu Ikeda.

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